

4/4/2016 الاثنين

م. خطاب

حاضرة [7]

State Space Approaches: -

- ① Representation      ② Analysis      ③ Design

For Linear Systems

$$\dot{x} = Ax + Bu \rightarrow f(x, u) \quad x: \text{state vector}$$

$$y = Cx + Du \quad u: \text{control input} ; y: \text{output}$$

A, B, C, D system Matrices

In Discrete Form

$$X((k+1)T) = A_d X(kT) + B_d U(kT)$$

$$Y(kT) = C_d X(kT) + D_d V(kT)$$

$$A_d \neq A ; C_d \neq C , B_d \neq B ; D_d \neq D \leftarrow$$

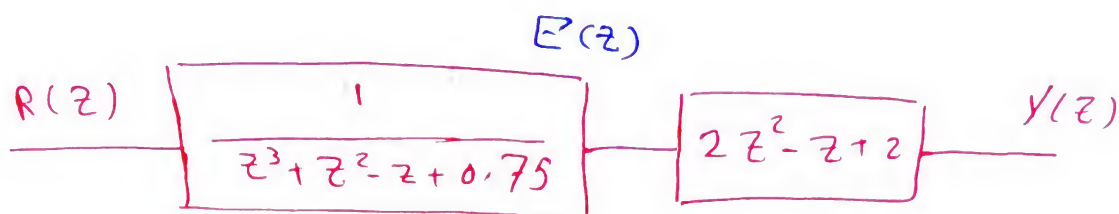
Representation of state variable Model:-

- ① Controller Canonical Form.
- ② Observer Canonical Form.
- ③ Parallel/Diagonal Form.
- ④ Cascaded Form.

Ex: Given  $\frac{Y(z)}{R(z)} = \frac{2z^2 - z + 2}{z^3 + z^2 - z + 0.75}$

Required

- ① Control canonical Form.
- ② observer Canonical Form.



1st Block

$$\frac{E(z)}{R(z)} = \frac{1}{z^3 + z^2 - z + 0.75}$$

$$\Rightarrow \text{inverse} \downarrow \quad z^3 E(z) + z^2 E(z) - z E(z) + 0.75 E(z) = R(z)$$

$$e(k+3) + e(k+2) - e(k+1) + 0.75 e(k) = r(k)$$

$$\text{Let } e(k) = x_1; \quad e(k+1) = x_2; \quad e(k+2) = x_3$$

$$\therefore e(k+1) = x_1(k+1); \quad e(k+2) = x_2(k+1); \quad e(k+3) = x_3(k+1)$$

$$\rightarrow e(k+3) = r(k) - e(k+2) + e(k+1) - 0.75 e(k)$$

$$x_1(k+1) = x_2(k)$$

$$x_2(k+1) = x_3(k)$$

$$x_3(k+1) = r(k) - x_3(k) + x_2(k) - 0.75 x_1(k)$$

$$X(k+1) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.75 & 1 & -1 \end{bmatrix} X(k) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(k)$$

2nd Block

$$\frac{Y(z)}{E(z)} = 2z^2 - z + 2$$

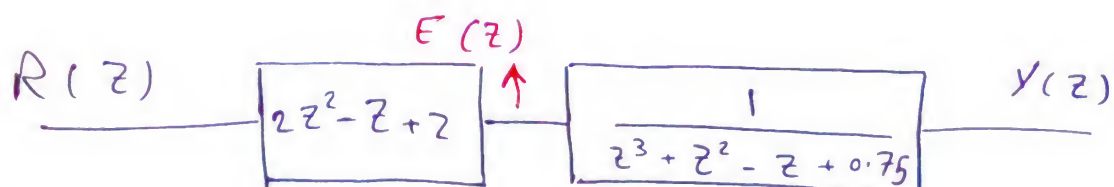
$$\text{inverse} \quad Y(z) = 2z^2 E(z) - z E(z) + 2 E(z)$$

$$\rightarrow y(k) = 2e(k+2) - e(k+1) + 2e(k)$$

$$\begin{array}{l} \text{Assumption: } x_1(k) = e(k) \\ \quad \quad \quad x_2(k) = e(k+1) \\ \quad \quad \quad x_3(k) = e(k+2) \end{array} \quad \left| \quad \begin{array}{l} y = 2x_3(k) - x_2(k) \\ \quad + 2x_1(k) \end{array} \right.$$

$$y(k) = \begin{bmatrix} 2 & -1 & 2 \end{bmatrix} x$$

\* Observer canonical form



1st block

$$\frac{E(z)}{R(z)} = 2z^2 - z + 2$$

$$\Rightarrow E(z) = 2z^2 R(z) - z R(z) + 2R(z)$$

inverse  $\Downarrow$

$$e(k) = 2r(k+2) - r(k+1) + 2r(k)$$

2nd Block

$$\frac{Y(z)}{E(z)} = \frac{1}{z^3 + z^2 - z + 0.75}$$

$$z^3 Y(z) + z^2 Y(z) - z Y(z) + 0.75 Y(z) = E(z)$$

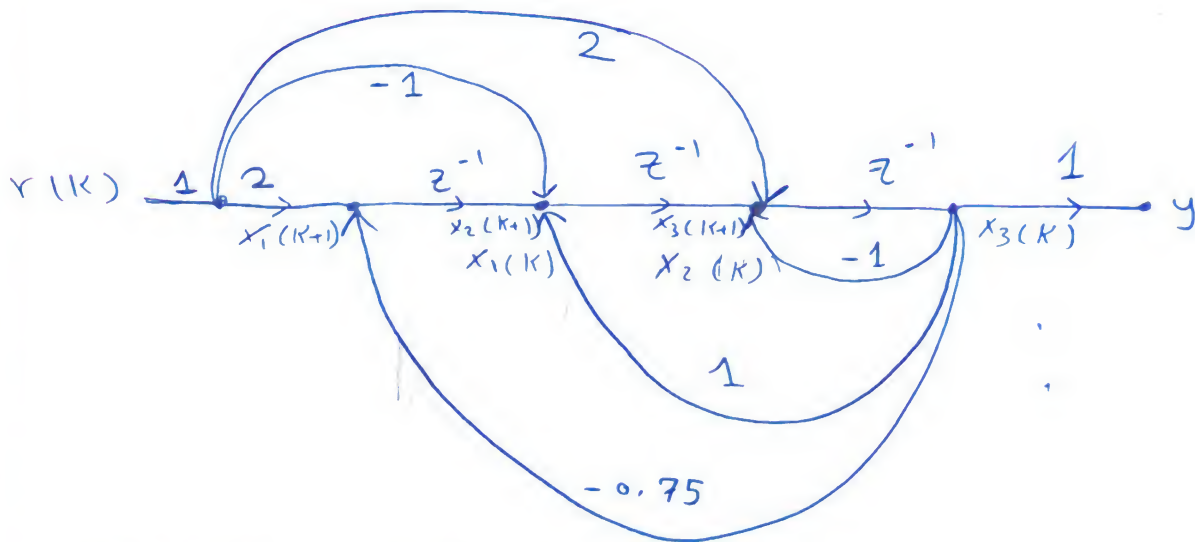
inverse  $\Downarrow$

$$\underbrace{y(k+3) + y(k+2) - y(k+1) + 0.75 y(k)}_{x_3(k+1)} = e(k)$$

$$\frac{Y(z)}{R(z)} = \frac{2z^2 - z + 2}{z^3 + z^2 - z + 0.75} * \frac{z^{-3}}{z^{-3}}$$

$$= \frac{2z^{-1} - z^{-2} + 2z^{-3}}{1 + z^{-1} - z^{-2} + 0.75z^{-3}}$$

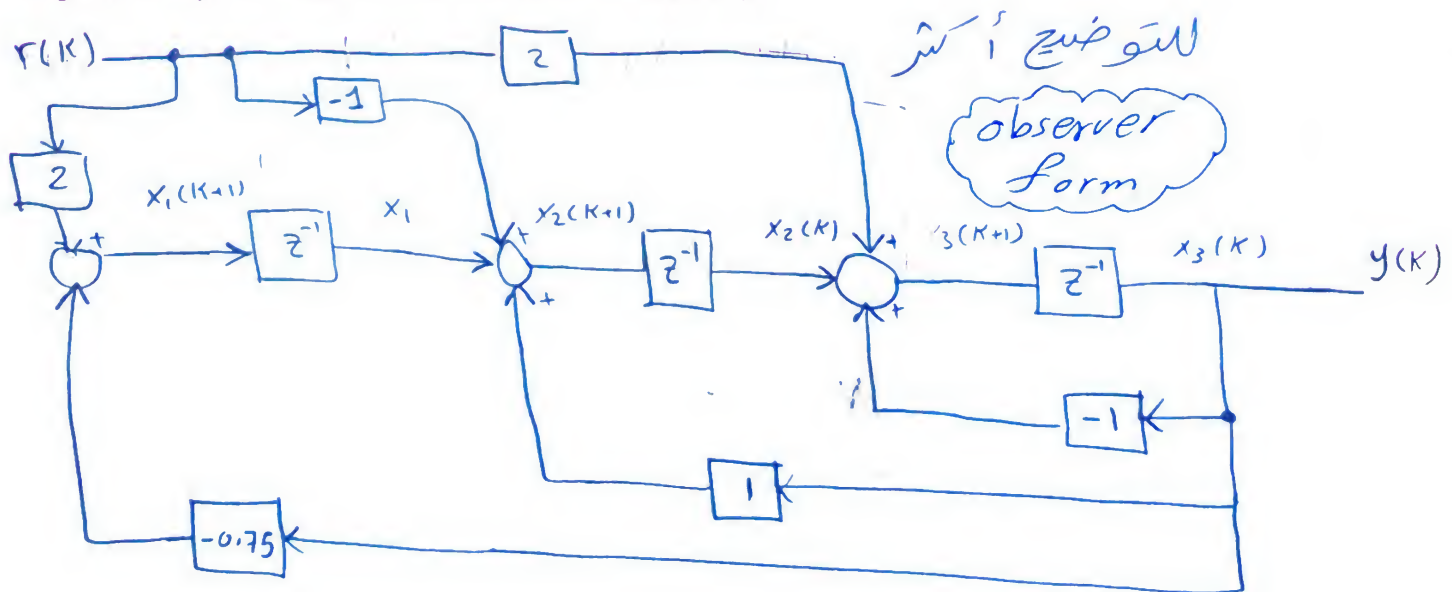
$$= \frac{2z^{-1} - z^{-2} + 2z^{-3}}{1 - [-z^{-1} + z^{-2} - 0.75z^{-3}]}$$



$$x_1(k+1) = -0.75 x_3(k) + 2 r(k)$$

$$x_2(k+1) = x_1(k) - r(k) + x_3(k)$$

$$x_3(k+1) = x_2(k) + 2 r(k) - x_3(k)$$

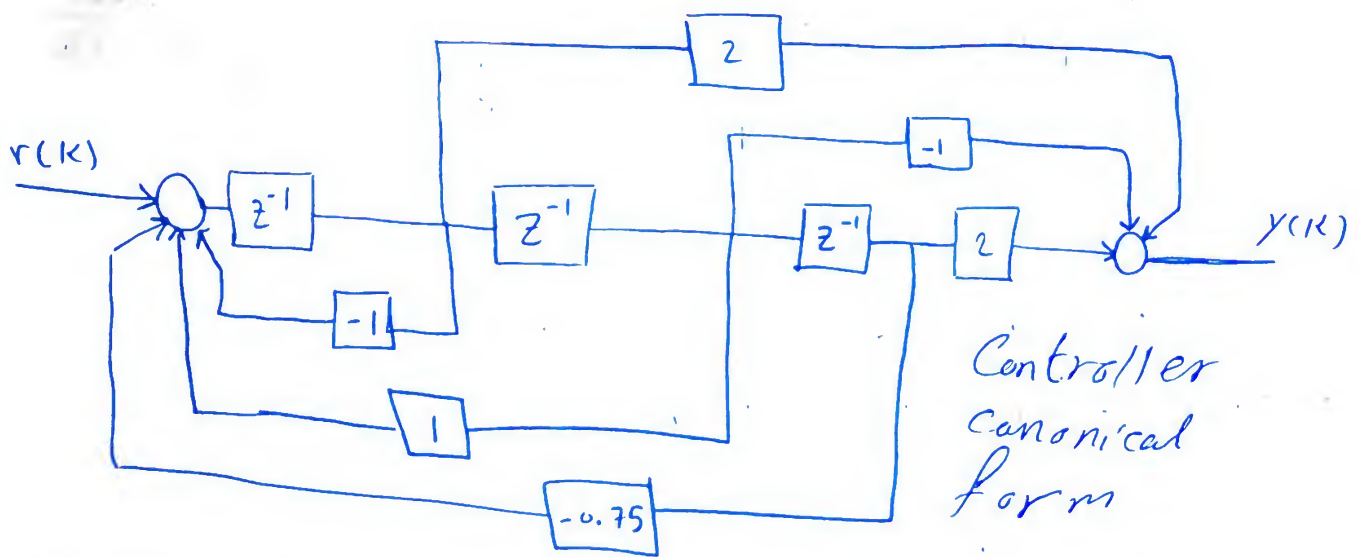


$$X(k+1) = \begin{bmatrix} 0 & 0 & -0.75 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} X(k) + \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} r(k)$$

$$y(k) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} X(k)$$

$$\frac{Y(z)}{R(z)} = \frac{\sum P_i}{1 - [\text{Loops}]}$$





\* Given T.F.

$$\frac{Y(z)}{R(z)} = \frac{z+0.5}{(z-1)(z-0.1)(z+0.4)}$$

- Required: Representation in diagonal form

$$T.F. = \frac{A}{z-1} + \frac{B}{z-0.1} + \frac{C}{z+0.4}$$

$$= \frac{1.19}{z-1} + \frac{-1.333}{z-0.1} + \frac{0.143}{z+0.4}$$

الأرقام لا تكون  
موجبة

$$Y(z) = \frac{1.19}{z-1} R(z) + \frac{-1.333}{z-0.1} R(z) + \frac{0.143}{z+0.4} R(z)$$

inverse  $\downarrow$   $\frac{Y_1}{1} = \frac{1.19}{z-1} R(z) \Rightarrow z Y_1(z) - Y_1(z) = 1.19 R(z)$

$$y_1(k+1) - y_1(k) = 1.19 r(k)$$

$$\text{Let } y_1(k) = x_1(k)$$

$$x_1(k+1) = y(k+1)$$

$$x_1(k+1) = x_1(k) + 1.19 r(k)$$

$$Y_2(z) = \frac{-1.33}{z-0.1} R(z)$$

$$z X_2(z) - 0.1 Y_2(z) = 1.33 R(z)$$

$$\Downarrow y_2(k+1) - 0.1 y_2(k) = 1.33 r(k)$$

$$\text{Let } x_2(k) = y_2(k) \rightarrow x_2(k+1) = y_2(k+1)$$

$$x_2(k+1) = 0.1 x_2(k) - 1.33 r(k)$$

$$Y_3(z) = \frac{0.143}{z+0.4} R(z)$$

$$z X_3(z) + 0.4 Y_3(z) = 0.143 R(z)$$

$$\Downarrow y_3(k+1) + 0.4 y_3(k) = 0.143 r(k)$$

$$\text{Let } x_3 = y_3$$

$$x_3(k+1) = y_3(k+1)$$

$$x_3(k+1) = -0.4 x_3(k) + 0.143 r(k)$$

$$X(k+1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & -0.4 \end{bmatrix} + \begin{bmatrix} 1.19 \\ -1.33 \\ 0.143 \end{bmatrix} r(k)$$

$$y(k) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} X(k)$$